Fonctions circulaires et hyperboliques

Propriétés trigonométriques *: remplacer* $\cos$ *par* ch *et* $\sin$ *par* $i.\operatorname{sh}$.

$$\cos(a+b) = \cos a.\cos b - \sin a.\sin b$$
$$\cos(a-b) = \cos a.\cos b + \sin a.\sin b$$
$$\sin(a+b) = \sin a.\cos b + \sin b.\cos a$$
$$\sin(a-b) = \sin a.\cos b - \sin b.\cos a$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a.\tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a.\tan b}$$

$$\cos 2a = 2.\cos^2 a - 1 = 1 - 2.\sin^2 a = \cos^2 a - \sin^2 a$$
$$\sin 2a = 2.\sin a.\cos a$$
$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

$$\cos a.\cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$
$$\sin a.\sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin a.\cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos p + \cos q = 2.\cos\frac{p+q}{2}.\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2.\sin\frac{p+q}{2}.\sin\frac{p-q}{2}$$
$$\sin p + \sin q = 2.\sin\frac{p+q}{2}.\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2.\sin\frac{p-q}{2}.\cos\frac{p+q}{2}$$

$$\operatorname{ch}(a+b) = \operatorname{ch} a.\operatorname{ch} b + \operatorname{sh} a.\operatorname{sh} b$$
$$\operatorname{ch}(a-b) = \operatorname{ch} a.\operatorname{ch} b - \operatorname{sh} a.\operatorname{sh} b$$
$$\operatorname{sh}(a+b) = \operatorname{sh} a.\operatorname{ch} b + \operatorname{sh} b.\operatorname{ch} a$$
$$\operatorname{sh}(a-b) = \operatorname{sh} a.\operatorname{ch} b - \operatorname{sh} b.\operatorname{ch} a$$
$$\operatorname{th}(a+b) = \frac{\operatorname{th} a + \operatorname{th} b}{1 + \operatorname{th} a.\operatorname{th} b}$$
$$\operatorname{th}(a-b) = \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a.\operatorname{th} b}$$

$$\operatorname{ch} 2a = 2.\operatorname{ch}^2 a - 1 = 1 + 2.\operatorname{sh}^2 a = \operatorname{ch}^2 a + \operatorname{sh}^2 a$$
$$\operatorname{sh} 2a = 2.\operatorname{sh} a.\operatorname{ch} a$$
$$\operatorname{th} 2a = \frac{2\operatorname{th} a}{1 + \operatorname{th}^2 a}$$

$$\operatorname{ch} a.\operatorname{ch} b = \frac{1}{2}\left[\operatorname{ch}(a+b) + \operatorname{ch}(a-b)\right]$$
$$\operatorname{sh} a.\operatorname{sh} b = \frac{1}{2}\left[\operatorname{ch}(a+b) - \operatorname{ch}(a-b)\right]$$
$$\operatorname{sh} a.\operatorname{ch} b = \frac{1}{2}\left[\operatorname{sh}(a+b) + \operatorname{sh}(a-b)\right]$$

$$\operatorname{ch} p + \operatorname{ch} q = 2.\operatorname{ch}\frac{p+q}{2}.\operatorname{ch}\frac{p-q}{2}$$
$$\operatorname{ch} p - \operatorname{ch} q = 2.\operatorname{sh}\frac{p+q}{2}.\operatorname{sh}\frac{p-q}{2}$$
$$\operatorname{sh} p + \operatorname{sh} q = 2.\operatorname{sh}\frac{p+q}{2}.\operatorname{ch}\frac{p-q}{2}$$
$$\operatorname{sh} p - \operatorname{sh} q = 2.\operatorname{sh}\frac{p-q}{2}.\operatorname{ch}\frac{p+q}{2}$$

avec $t = \tan\dfrac{x}{2}$ $\begin{cases} \cos x &= \frac{1-t^2}{1+t^2} \\ \sin x &= \frac{2t}{1+t^2} \\ \tan x &= \frac{2t}{1-t^2} \end{cases}$

avec $t = \mathrm{th}\,\dfrac{x}{2}$ $\begin{cases} \mathrm{ch}\, x &= \frac{1+t^2}{1-t^2} \\ \mathrm{sh}\, x &= \frac{2t}{1-t^2} \\ \mathrm{th}\, x &= \frac{2t}{1+t^2} \end{cases}$

Dérivées *: la multiplication par i n'est plus valable*

$$\cos' x = -\sin x$$
$$\sin' x = \cos x$$
$$\tan' x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$
$$\mathrm{cotan}' x = -1 - \mathrm{cotan}^2 x = \frac{-1}{\sin^2 x}$$

$$\mathrm{ch}' x = \mathrm{sh}\, x$$
$$\mathrm{sh}' x = \mathrm{ch}\, x$$
$$\mathrm{th}' x = 1 - \mathrm{th}^2 x = \frac{1}{\mathrm{ch}^2 x}$$
$$\mathrm{coth}' x = 1 - \mathrm{coth}^2 x = \frac{-1}{\mathrm{sh}^2 x}$$

$$\mathrm{Arccos}' x = \frac{-1}{\sqrt{1-x^2}} \quad (|x| < 1)$$
$$\mathrm{Arcsin}' x = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$$
$$\mathrm{Arctan}' x = \frac{1}{1+x^2}$$
$$\mathrm{Arccotan}' x = \frac{-1}{1+x^2}$$

$$\mathrm{Argch}' x = \frac{1}{\sqrt{x^2-1}} \quad (x > 1)$$
$$\mathrm{Argsh}' x = \frac{1}{\sqrt{x^2+1}}$$
$$\mathrm{Argth}' x = \frac{1}{1-x^2} \quad (|x| < 1)$$
$$\mathrm{Argcoth}' x = \frac{1}{1-x^2} \quad (|x| > 1)$$